THE RADIX.

A NEW WAY OF MAKING LOGARITHMS.

This RULE, by One Hundred LOGARITHMS, constructs the LOGARITHMS to all Numbers, from 1 to 100000000000000000000.

To TWENTY Places of Figures in each LOGARITHM.

AND ALSO

Numbers are found from the LOGARITHMS, to the same Length of TWENTY Places.

After a most concise and easy Manner.

WITH

Their Application to the Involution of POWERS, Extracting of ROOTS, &c. to the Extent of TWENTY Figures.

In FIVE PROBLEMS.

By ROBERT FLOWER.

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M.DCC.LXXI.

TO

GEORGE JACKSON, Esq;

DEPUTY SECRETARY of the ADMIRALTY.

SIR,

KNOWING you to be a Gentleman of Ability to judge of the Merit of a Performance calculated to improve useful Science; to which End nothing occurs to me as more conducive, than an easy and expeditious Rule for finding Logarithms to Numbers, and Numbers to Logarithms given; I persuade myself, invited by that generous and publick-spirited Sentiment which you have always manifested; that you will permit this New Work, intended for the above Purposes, to make it's first Appearance under your Protection.

The

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6

The Method here laid down is plain, eaſy and conciſe, founded in Numbers; without the Hyperbola or any other mathematical Deduction: On the contrary, I have ſhewn how to deduce an Hyperbolical Logarithm from this Rule.

As the ſame Rule that makes Logarithms to Numbers, conſtructs alſo Logarithms to the Natural Sines, Tangents, Secants, &c. it cannot fail to be uſeful at Sea, to all Gentlemen that make uſe of the TABLES.

I am,

SIR,

Your moſt humble

and moſt obedient Servant,

ROBERT FLOWER.

TO

The READER.

THE Construction of Logarithms having been hitherto accounted a Work of considerable Labour, as well as Difficulty, and as the Use of these Artificial Numbers in the Mathematicks is acknowledged, there have been many Ways invented to facilitate the Operation, in order to make Logarithms to more Places of Figures, and to larger Numbers than are found in the Tables, or can be made by them.

The following Method is supposed to be the shortest and easiest known at present, for finding Logarithms from Numbers and Numbers from Logarithms to Twenty Places of Figures: by an Instrument or RULE, calculated for that Purpose, called the RADIX, consisting

consisting of *One Hundred Numbers* and their *Logarithms*; viz. the Resolvend Number 10. whose Logarithm is given, and Ninety-nine Classical Roots of Number 10. whose Logarithms (except the Logarithm of 1.) are constructed to Twenty-one Places in Decimals, having 0. for the common Index, as Numbers between 1. and 10.

This Instrument I explain as follows; viz. The uppermost Division contains the Number 10. and it's Logarithm 1. The next lower Division (0) shews the Nine Digits in Units Place, with their Logarithms. The next lower Division (1) exhibits the Nine Digits in the *First* Place of Decimals, with their Logarithms. The Division (2) shews the Nine Digits in the *Second* Place of Decimals, with their Logarithms: and so on to the Tenth; with all which Classes of the Nine Digits, the Large Figure 1 standing alone in the First Column is understood to be connected: So that the Numbers expressed at large (except Units Place) are, viz. *First* Class, 1.9 1.8 1.7 1.6 1.5 1.4 1.3 1.2 1.1 —— *Second* Class, 1.09 1.08 1.07 1.06 1.05 1.04 &c. —— *Third* Class, 1.009 1.008 1.007

1.006

1.006 &c.——*Fourth* Claſs, 1.0009 1.0008 1.0007 1.0006 &c. down to the Tenth Claſs, numbered 0. 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. in the Second Column on the RULE, which Figures ſhew reſpectively the decimal Place, which each Claſs of the Nine Digits ſtands in: Alſo the Head of each Logarithm in the Fourth Column, and it's Number in the Third Column, are always in the ſame Place of Decimals.

In the following Work I have introduced a New Way of Approximation, in making Logarithms, by the Cube Root, that is, by multiplying and dividing continually with Cube Roots of 10. which Roots being all Compound Numbers; therefore the Multiplications and Diviſions become more operoſe, the further ſuch Roots are extracted. Now Square Roots, Cube-Square Roots, or even Surſolid Roots of 10, will alſo effect the ſame thing, and in the ſame compound manner.

But Multiplication by a Claſſic Root of 10. is always done in One Line of Figures, though it be extended to Twenty Places, anſwering to a Compound Multiplication of Twenty Lines. Thus far theſe

Claſſic

Classic Numbers exceed Compound Numbers: Add to this their converging Power, which again cuts off half those so short Multiplications.

This Method is ready to apply. When a Logarithm is to be made, we proceed immediately from the Number, by easy steps, to get the Logarithm: And with equal facility, and without a moment's loss, we proceed from a Logarithm to find the Number.

From these Principles an expeditious Way of constructing Logarithms to Natural Numbers, Natural Sines, Tangents, Secants, &c. is deduced. As also an easy and uniform Rule for involving Powers and extracting Roots to Twenty Places of Figures: and consequently Compound Interest for Years and Days, &c. are carried to a much greater height and exactness by this New portable RULE, than by the best TABLES now extant.

THE

R A D I X.

A

New Way of making Logarithms.

PROBLEM I.

A new Method of Constructing Logarithms by the Cube Root; intended here as an INTRODUCTION only.

DEFINITION.

IN a SCALE of Natural Numbers and Logarithms, Number 10. is the Resolvend of the whole Scale: all numbers under ten are it's roots; all numbers above ten are it's powers; and the Logarithms are the Indices of the Resolvend, shewing what power or root of ten each number is,

Thus $10^0 = 1$; $10^1 = 10$; $10^2 = 100$; $10^3 = 1000$; $10^4 = 10000$; $10^5 = 100000$; &c. Whence, taking away the Resolvend,

Numb.	Log.
1	0.
10	1.
100	2.
1000	3.
10000	4.
100000	5.
&c.	&c.

These Indices 0. 1. 2. 3. 4. 5. &c. which are the Logarithms of the Natural Numbers 1. 10. 100. 1000. 10000. 100000. &c. are given to find the Logarithms of all Intermediate numbers in the Scale.

 This

This FIRST Construction though a Work of great labour, as the roots of 10. by which we multiply and divide are COMPOUND Numbers; yet neverthelefs it converges much fafter than the ancient way by the Mean Proportional.

A

Cube RADIX of 10.

10.	=	$r^{10460353203}$
2.1544346900	=	$r^{3486784401}$
1.2915496651	=	$r^{1162261467}$
1.0890229623	=	$r^{387420489}$
1.0288348789	=	$r^{129140163}$
1.0095206950	=	$r^{43046721}$
1.0031635464	=	$r^{14348907}$
1.0010534054	=	$r^{4782969}$
1.0003510119	=	$r^{1594323}$
1.0001169903	=	r^{531441}
1.0000389952	=	r^{177147}
1.0000129982	=	r^{59049}
1.0000043327	=	r^{19683}
14442	=	r^{6561}
04814	=	r^{2187}
01604	=	r^{729}
00534	=	r^{243}
00178	=	r^{81}
00059	=	r^{27}
00019	=	r^{9}
00006	=	r^{3}
00002	=	r

This RADIX of twenty-one Cube Roots, whereof twelve are got by Extraction, and nine by dividing by 3; comprehends a Series of 10460353203 Lines, or Terms in Geometrical Progreffion r. r^2. r^3. r^4. r^5. r^6. r^7. r^8. r^9. &c. to $r^{10460353203}$, of eleven figures each; any one of which Terms, fhewing the value of a power propofed r^n, may be produced two ways from it. And on the contrary,

This RADIX reverfed, comprehends by approximation a Table of Natural Numbers in Arithmetical Progreffion 1. 2. 3. 4. 5. 6. 7. 8.

9.

9. 10. 11. 12. &c. to 10000000000, and two Tables of Logarithms to the Numbers; any one of which Numbers with its two Logarithms at one approximation, may be produced from it two different ways.

To approximate any Number and conſtruct its Logarithm.

Rule. Put down the given Number as a root of ten, then ſelect ſome root of ten as near it as you can, which if too little multiply continually till it becomes too much; then divide continually till it becomes too little, &c. alternately, by leſs and leſs roots of ten; till you have approximated the NUMBER to as many places, as you intend to have figures in the Logarithin. Conſequently the Indices of the diviſors ſubtracted from the addition of the other Indices, as you proceed, will contruct the Index or LOGARITHM.

EXAMPLE I.

To approximate the Number 2, and conſtruct it's Logarithm.

$$
\begin{array}{rcl}
2.1544346900 & = & r^{3486784401} \\
1.0890229623 & = & r^{-387420489} \\
\hline
1.9783188827 & = & r^{3099363912} \\
1.0095206950 & = & r^{43046721} \\
\hline
1.9971538535 & = & r^{3142410633} \\
1.0010534054 & = & r^{4782969} \\
\hline
1.9992576662 & = & r^{3147193602} \\
1.0003510119 & = & r^{1594323} \\
\hline
1.9999594295 & = & r^{3148787925} \\
1.0000129982 & = & r^{59049} \\
\hline
1.9999854254 & = & r^{3148846974} \\
1.0000043327 & = & r^{19683} \\
\hline
1.9999940907 & = & r^{3148866657} \\
1.0000043327 & = & r^{19683} \\
\hline
2.0000027561 & = & r^{3148886340} \\
1.0000014442 & = & r^{-6561} \\
\hline
1.9999998677 & = & r^{3148879779} \\
1\ 0000000534 & = & r^{243} \\
\hline
\end{array}
$$

$$1.9999999745 = r^{3148880022}$$
$$1.0000000178 = r^{81}$$
$$2.0000000101 = r^{3148880103}$$
$$1.0000000059 = r^{-27}$$
$$1.9999999984 = r^{3148880076}$$
$$1.0000000006 = r^{3}$$
$$1.9999999996 = r^{3148880079}$$
$$1.0000000002 = r^{1}$$
$$2.0000000000 = r^{3148880080}$$

In the RADIX I find the nearest root of ten to the given Number 2. is 2.1544346900, which being too much I divide by 1.0890229623 and it quotes 1.9783188827; this being too little I multiply by 1.0095206950 and it produces 1.9971538535; which being too little I multiply by 1.0010534054 and it produces 1.9992576662 ——— being too little I multiply by 1.0003510119 and it produces 1.9999594295;—being too little I multiply by 1.0000129982 and it produces 1.9999854254—being too little I multiply by 1.0000043327 and it produces 1.9999940907——— being too little I multiply by 1.0000043327 and it produces 2.0000027561; which being too much I divide by 1.0000014442 and it quotes 1.9999998677; which being too little I multiply as above, &c. till I get the Number 2. with ten ciphers in the decimal, by which I count that the Index which always keeps pace with the Number has in it ten true figures; viz. $2.0000000000 = r^{3148880080}$.

But the Index of the root divided by the denominator of the root shews the Index of the Resolvend.

$$\text{Thus } 2. = r^{3148880080} = 10^{\frac{3148880080}{10460353203}} = 10^{0.3010299957}.$$

Consequently 0.3010299957 is the Logarithm of 2. As by the Definition.

In this manner we get two Logarithms, to every Number, at a time.

Number.	Radical Log.	Resolvend Log.
2. ———	3148880080 ———	0.3010299957

Both

Both these kinds of Logarithms will anſwer the ſame purpoſes, in multiplication, diviſion, involution, evolution, &c. But the Radical Logarithm alſo ſhews what place in the immenſe Series, $r. r^{2}. r^{3}. r^{4}. r^{5}. r^{6}. r^{7}.$ &c. up to any height any Number poſſeſſes: thus by approximation the Number 2 is found to be $r^{3148880080}$ that is the 3148880080th Term of the above Series; And further, the Number 20 is the (3148880080 + 10460353203) 13609233283rd Term; And 200 is the (3148880080 + 10460353203×2) 24069586486th Term: And 2000 is the (3148880080 + 10460353203×3) 34529939689th Term. That is, the Radical Logarithm of any approximated Number, plus the Radical Characteriſtic of 10. 100. 1000. &c. will ſhew the place, which is ſhifting and not always the ſame, for every cube root leſs or more in the RADIX, will new form the Series, and change the place continually.

This Radical Logarithm on the other hand ſhews how to produce any Term of the Series demanded; as for inſtance, would you know the value of the 35690267362nd Term of the above Series of Numbers, it is to be found by the Indices of r, by adding and ſubtracting them (as hereunder) till you get the Index required——But becauſe the given Index 35690267362 is greater than the characteriſtic of 10, it muſt be divided thereby to know how many Integers are to be in the Term. Thus,

$$\begin{array}{rl} 10460353203) & 35690267362 \text{ (3, that is, } 10^{3}\text{, characteriſtic of 1000.} \\ & \underline{31381059609} \\ & 4309207753 \end{array}$$

Find the remaining Index by the Indices of the RADIX, thus,

$$r^{3486784401 + 1162261467 - 387420489 + 43046721 + 4782969 - 177147 - 59049 - 19683 + 6561 + 2187 - 243 + 81 - 27 + 3 + 1}$$

$= r^{4309207753} = 2.58200000002$; which Number I produced by multiplying and dividing the Radical Number, $r^{248678440}$ viz. Number 2.1544346900 continually by the roots correſpondent to the above + and — Indices. But 2.582, &c. × 1000 = 2582. The Term ſought, which appears to be a Whole Number. And this is the way to find a natural Number belonging to a Radical Logarithm. Hence riſes a Queſtion,

Why should Number 2, possess the 3148880080 place in the Scale of Numbers, when Log. 3010299957 is said to be the place? Now what seems here to confute the Theory, establishes it. Because in the former Series the Number of Terms is 10460353203: But in the latter Series the Number of Terms is but 10000000000. Therefore the place is changed. For,

As 10460353203 : 3148880080 : : 10000000000 : 3010299957.

EFFECTION 2.

To construct decimally the Resolvend Logarithms of 10. By substituting the Indices of 10. instead of the Indices of *r*.

10.	=	1.
2.1544346900	=	0.3333333333
1.2915496651	=	0.1111111111
1.0890229623	=	0.0370370370
1.0288348789	=	0.0123456790
1.0095206950	=	0.0041152263
1.0031635464	=	0.0013717421
1.0010534054	=	0.0004572474
1.0003510119	=	0.0001524158
1.0001169903	=	0.0000508053
1.0000389952	=	0.0000169351
1.0000129982	=	0.0000056450
1.0000043327	=	0.0000018817
14442	=	6272
4814	=	2091
1604	=	697
534	=	232
178	=	77
59	=	26
19	=	9
6	=	3
2	=	1

Here the roots are the same as before, extracted till half the decimal is ciphers, and then the rest are got by dividing the decimal continually

continually by 3. The decimal Indices are also found by dividing 1. continually by 3.

EXAMPLE II.

To approximate the Number 2. and construct its Resolvend Logarithm.

	2.1544346900	=	0.3333333333	
	1.0890229623	=	0.0370370370	
	1.9783188827	=	0.2962962963	
	1.0095206950	=	0.0041152263	
	1.9971538535	=	0.3004115226	
	1.0010534054	=	0.0004572474	
	1.9992576662	=	0.3008687700	
	1.0003510119	=	0.0001524158	
	1.9999594295	=	0.3010211858	
	1.0000129982	=	0.0000056450	
	1.9999854254	=	0.3010268308	
	1.0000043327	=	0.0000018817	
	1.9999940907	=	0.3010287125	
	1.0000043327	=	0.0000018817	
	2.0000027561	=	0.3010305942	
	1.0000014442	=	0.0000006272	
	1.9999998677	=	0.3010299670	
	1.0000000534	=	0.0000000232	
	1.9999999745	=	0.3010299902	
	1.0000000178	=	0.0000000077	
	2.0000000101	=	0.3010299979	
	1.0000000059	=	0.0000000026	
	1.9999999984	=	0.3010299953	
	1.0000000006	=	0.0000000003	
	1.9999999996	=	0.3010299956	
	1.0000000002	=	0.0000000001	
Number	2.0000000000	=	0.3010299957	Logarithm.

Here

Here the approximation of the Number 2 is the same as in the preceding; and this Logarithm is the very same as that Resolvend Logarithm constructed there from the Radical one.

The Reverse rule is when the Resolvend Number 10. begins the Approximation.

Square RADIX of 10.

10.	=	1.
3.1622776602	=	0.5
1.7782794100	=	0.25
1.3335214321	=	0.125
1.1547819846	=	0.0625
1.0746078282	=	0.03125
1.0366329283	=	0.015625
1.0181517217	=	0.0078125
1.0090350448	=	0.00390625
1.0045073643	=	0.001953125
1.0022511483	=	0.0009765625
1.0011249414	=	0.0004882813
1.0005623126	=	0.0002441406
1.0002811167	=	0.0001220703
1.0001405484	=	0.0000610352
1.0000702717	=	0.0000305176
1.0000351352	=	0.0000152588
1.0000175674	=	0.0000076295
1.0000087836	=	0.0000038147
1.0000043918	=	0.0000019073
21959	=	9537
10979	=	4768
5489	=	2384
2744	=	1192
1372	=	596
686	=	298
343	=	149
171	=	75
85	=	37
42	=	19
21	=	9
10	=	5
5	=	2
2	=	1

By

By this RADIX of Square Roots continued (nineteen of which I extracted to ten places in decimals; and the rest I have by dividing the decimal, half ciphers, by 2) I prove the work done by the Cube RADIX; both producing the same thing in the same manner, by multiplying and dividing by the roots, and by adding and substracting the Logarithms. Now this having Square roots more than the other has cube roots; the Intervals here are less than the Intervals between the cube roots, and therefore it happens that the Numbers are approximated somewhat sooner by the Square than by the Cube: but these two may be used together as one RADIX, and then the Intervals will be much less, and consequently the approximation thereby greatly accelerated.

It was the consideration of these smaller Intervals that led me to the Classical RADIX (which I constructed to ten decimal places both ways) where the Intervals between the roots are regular; decreasing by an Unit at a time in every figure of the decimal.

Classical RADIX of 10.

——For Practice it would be convenient to have the RADIX impressed on a portable Box RULE, of this form.

Here Unity in the first column, standing alone, connects with all the Radical Numbers as multipliers and divisors: Or it is the First Term common to all when considered as Ratios. (Except Units).

The Figures 0. 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. in the second column, shew the number of the Class, and which is the same thing, the place of decimals that each Class stands in.

The third column begins with the nine Digits in Units place (0). The rest are the decimal figures (without the ciphers) of the ten Classes.

The fourth column contains the Logarithms to the classic figures or roots opposed to them.

By this RULE (which may be made for the Pocket) one may construct a Logarithm in ten Minutes, often in seven, and sometimes in three or four Minutes.

The RADIX.

N.	10.		Log. 1.
		9	.9542425094
		8	.9030899870
		7	.8450980400
		6	.7781512504
	0	5	.6989700043
		4	.6020599913
		3	.4771212547
		2	.3010299957
		1	.0000000000
		9	.2787536010
		8	.2552725051
		7	.2304489214
		6	.2041199827
	1	5	.1760912591
		4	.1461280357
		3	.1139433523
		2	0791812460
		1	.0413926852
		9	.0374264979
		8	0334237555
		7	.0293837777
		6	.0253058653
	2	5	.0211892991
		4	.0170333393
		3	0128372247
		2	.0086001718
		1	0043213738
		9	.0038911662
		8	.0034605321
		7	.0030294706
		6	.0025979807
	3	5	.0021660618
		4	.0017337128
		3	.0013009330
		2	.0008677215
		1	.0004340775
		9	.0003906892
		8	.0003472967
		7	.0003038998
		6	.0002604985
	4	5	.0002170930
		4	.0001736831
		3	0001302688
		2	.0000868502
		1	.0000434273
		9	0000390847
		8	.0000347422
		7	0000303995
		6	.0000260569

N.			Log.
	5	5	.0000217142
		4	.0000173714
		3	.0000130286
		2	.0000086858
		1	.0000043429
1		9	.0000039086
		8	.0000034743
		7	.0000030401
		6	.0000026058
	6	5	.0000021715
		4	.0000017372
		3	.0000013029
		2	.0000008686
		1	.0000004343
		9	.0000003909
		8	.0000003474
		7	.0000003040
		6	.0000002606
	7	5	.0000002171
		4	.0000001737
		3	.0000001303
		2	.0000000869
		1	.0000000434
		9	.0000000391
		8	.0000000347
		7	.0000000304
		6	.0000000261
	8	5	.0000000217
		4	.0000000174
		3	.0000000130
		2	.0000000087
		1	0000000043
		9	.0000000039
		8	.0000000035
		7	.0000000030
		6	.0000000026
	9	5	.0000000022
		4	0000000017
		3	.0000000013
		2	.0000000009
		1	.0000000004
		9	.0000000004
		8	.0000000004
		7	.0000000003
		6	.0000000003
	10	5	.0000000002
		4	.0000000002
		3	.0000000001
		2	.0000000001
		1	0000000000

PROBLEM II.

To construct the Logarithm of any Number to Ten places of Figures, by the *Right Rule*, that approximates from a Number in the RADIX to the Given Number.

Multiplication by a Classical Number is always the Operation of a Single Figure, and done in one line of Figures; effecting the same thing in that one line, that is done by Compound Multiplication or Division, by Square roots, Cube roots, &c. in ten or twenty lines: So that here the Approximation or Work of the Numbers is as short as that of the Logarithms.

This is called the *Right Rule* not only because the Approximation is made the right way, from a Number whose Logarithm is known, to a Number whose Logarithm is unknown; but because the Signs + and — are applied right; that is, Multiplication in Numbers corresponds with Addition in the Logarithms, and Division in Numbers with Subtraction in the Logarithms. Also because the Numbers and Logarithms every where stand right against one another.

The Logarithms made use of in this Problem are all to be found in the RADIX preceding, by referring to the Classic Number; that is, to the same decimal place numbered in the second Column, and to the same Figure put down in the third Column.

This RADIX consisting of ninety classical roots of 10, whose Intervals are units, together with the nine digits and Number 10, making in the whole 100 Numbers, produces by multiplication and division, all Numbers up to 10000000000. two ways; and their 100 Logarithms construct, by addition and subtraction, the Logarithms of those Numbers, to ten figures in the Logarithm, two ways.

The Direct rule is when the roots of 10 are used alone; and the Reverse rule is when the Resolvend 10, is also used.

The first thing to be observed in the approximation of a Number is the highest figure, which is always to be found amongst the nine digits, from which figure you may by multiplying the approximation when it is too little, and dividing when it is too much, by the classical numbers, with ease, approximate any Number: And by adding the Logarithms of the multipliers, and subtracting the Logarithms of the divisors, you produce the Logarithm.

EXAMPLE I.

To approximate the Number 9876543210, and construct it's Logarithm, by the Direct rule; that is, without the Number 10.

	9.	=	0.9542425094
	1.09	=	+ 374264979
	9.81	=	0.9916690073
	1.006	=	+ 25979807
	9.86886	=	0.9942669880
	1.0007		+ 3038998
	9.875768202	=	0.9945708878
	1.00007		+ 303995
	9.8764595058	=	0.9946012873
	1.000008		+ 34743
	9.8765385175	=	0.9946047616
	1.0000004		+ 1737
	9.8765424681	=	0.9946049353
	1.00000007		+ 304
	9.8765431595	=	0.9946049657
	1.000000005		+ 22
	9.8765432089	=	0.9946049679
	1.0000000002		+ 1
Number	9.8765432109	=	0.9946049680 Logarithm.

Here I put down the highest figure in the given Number as a root of ten; viz. 9. which being too little I multiply by 1.09 in one line, by adding Nine-times 9. two places lower to itself, which produces 9.81; This being too little, I multiply by 1.006 by adding Six-times 9.81 three places lower to itself, which produces 9.86886; This being too little, I multiply by 1.0007 by adding Seven-times 9.86886 four places lower to itself, which produces 9.875768202; &c. (as above) till it produces the Number: And adding the Logarithms of the Multipliers continually to the Logarithm of 9. produces it's Logarithm.

Note.

Note. These Logarithms, which for plainness are added one by one, might be set down all together, and added into one Sum.

To approximate Number 9876543210, and construct it's Logarithm, by the Reverse Rule, dividing Number 10.

	10.	=	1.0000000000	
	9.	−	.9542425094	
	1.1111111111	=	0.0457574906	
	8.	+	.9030899870	
	8.8888888888	=	0.9488474776	
	1.1	+	413926852	
	9.7777777777	=	0.9902401628	
	1.01	+	43213738	
	9.8755555555	=	0.9945615366	
	1.0001	+	434273	
	9.8765431111	=	0.9946049639	
	1.00000001	=	+ 43	
Number	9.8765432099	=	0.9946049682	Logarithm.

First I put down 10. (the Resolvend) which being too much I divide by 9. thus $\frac{10}{9} = 1.1111111111$, &c. This being too little, I multiply by 8. and it produces 8.8888888888, &c. which being too little I multiply by 1.1 by adding Once the Multiplicand one place lower to itself, which produces 9.7777777777, &c. being too little I multiply by 1.01 by adding Once the Multiplicand two places lower to itself, which produces 9.8755555555, &c. being too little, I multiply by 1.0001 by adding the Multiplicand four places lower to itself, which produces 9.8765431111, &c. &c. Then, having put down the Logarithm of 10. I subtract the Logarithm of the divisor, and add the Logarithms of the Multipliers, whereby the Logarithm in the approximation keeps pace with the Number.

EXAMPLE II.

To approximate the Number 1234567890, and construct it's Logarithm, by the Direct Rule, that is, by the Roots of 10.

	1.2	=	0.0791812460	
	1.02	+	86001718	
	1.224	=	0.0877814178	
	1.008	+	34605321	
	1.233792	=	0.0912419499	
	1.0006	+	2604985	
	1.2345322752	=	0.0915024484	
	1.00002	+	86858	
	1.2345569658	=	0.0915111342	
	1.000008	+	34743	
	1.2345668423	=	0.0915146085	
	1.0000008	+	3474	
	1.2345678300	=	0.0915149559	
	1.00000004	+	174	
	1.2345678794	=	0.0915149733	
	1.000000009	+	39	
Number	1.2345678905	=	0.0915149772	Logarithm.

Here I put down the highest figures in the given Number; viz. 1.2 as a root of ten; which I multiply by 1.02 and it produces 1.224; this I multiply by 1.008 and it produces 1.233792, &c. Thus by continual multiplying the given Number is produced: And by continual addition the Logarithm is constructed.

To approximate Number 1234567890, and construct it's Logarithm, by the Reverse Rule, by dividing Number 10.

10.	=	1.0000000000
9.	—	.9542425094
1.1111111111	=	0.0457574906
1.1	+	413926852

1.2222222222

	1.2222222222	=	0.0871501758	
	1.01		+ 43213738	
	1.2344444444	=	0.0914715496	
	1.0001		+ 434273	
	1.2345678888	=	0.0915149769	
	1.000000001	=	+ 4	
Number	1.2345678900	=	0.0915149773	Logarithm.

Here dividing 10. by 9. and then multiplying the quotient continually by such classical numbers as upon trial appear to accelerate the approximation, produces the given Number: And adding the Logarithms of the multipliers continually to the Logarithm of 10. minus the Logarithm of the divisor, produces the Logarithm required.

Note 1. The chief difficulty here is to get the Multiplier: For if we put the approximation $= a$; and the Number approximated to $= n$; Then the value of the Multiplier is the First Figure of $\frac{n-a}{n}$; which when n is a compound number is tedious.

Note 2. But if n be $= 1$; Then the Multiplier is the First Figure of $\frac{1-a}{1}$; viz. The First Figure of the Arithmetical Complement, which is momentary.

Example III.

To construct the Logarithms of Number 9876543210, and Number 1234567890, the shortest way.

Direct Rule.

	8.		+ 0.9030899870	
	÷ 9.		— 0.9542425094	
	8.8888888888	=	0.9488474776	
	÷ 9.		— 0.9542425094	
Number	9.8765432099	=	0.9946049682	Logarithm.

Reverse

Reverse Rule.

	10.	+	1.
÷	9.	—	0.9542425094
	1.1111111111	+	0.0457574906
÷	9.	= —	0.9542425094
	1.2345679012	=	0.0915149812
÷	1.000000009	—	39
Number	1.2345678901	=	0.0915149773 Logarithm.

Here the divisors, dividends and quotients are all put down as roots of 10. and the Logarithms, as Logarithms of such roots.

COROLLARY.

Hence it appears that Twice the Logarithm of 9. subtracted from the Logarithm of 8. gives the Logarithm of 9876543210. and, that Twice the Logarithm of 9. plus the Logarithm of nine in the ninth Class, subtracted from the Logarithm of 10. gives the Logarithm of 1234567890. Both which Logarithms may be taken from the RADIX by Inspection.

Example IV.

To find the Number belonging to the Log. 0.3010299957.

0.3010299957		
0.2787536010	=	1.9
+ 211892991	=	1.05
0.2999429001	=	1.995
+ 8677215	=	1.002
0.3008106216	=	1.998990
+ 2170930	=	1.0005
0.3010277146	=	1.999989495
+ 21715	=	1.000005
0.3010298861	=	1.9999994949
+ 869	=	1.0000002

0.3010299730

	0.3010299730	=	1.9999998949	
	+ 217	=	1.00000005	
	0.3010299947	=	1.9999999949	
	+ 9	=	1.000000002	
	0.3010299956	=	1.9999999989	
	+ 2	=	1.0000000005	
Logarithm.	0.3010299958	=	1.9999999999	Numb. 2.

First I ſet down the Given Logarithm, in order to compare with the approximation. Then I take from the Radix Log. 0.2787536008 (the neareſt under the Given Logarithm) and it's Number 1.9; I then add continually ſuch Logarithms as approximate to the Given Logarithm: And multiply continually by their claſſical Numbers which approximate to the Number.

This is a General Rule, by which the Number belonging to any Logarithm may be found.

PROBLEM III.

To conſtruct the Logarithm of any Number, to Twenty places of Figures; by the *Reflected Rule*, which approximates from the Given Number, to a Number in the RADIX.

In all Radical Numbers whatever between 1. and 2. Unity is the Firſt Term, and the Decimal is the Negative Ratio of an Infinite Series of Geometrical Proportionals.

This is called the *Reflected Rule*, becauſe the Approximation is turned the contrary way; viz. from a Number whoſe Logarithm is unknown to a Number whoſe Logarithm is known, whereby the Signs + and — are reflected; viz. Diviſion in Numbers anſwers to Addition in the Logarithms, and Multiplication in Numbers, to Subtraction in the Logarithms; Alſo the place of Number and Logarithm is reflected.

This *Turn* is given to the Approximation, in order to get the Line of Ratios by the Arithmetical Complement (Note 2. Prob. 2.) as thus 7 from nine is 2; 5 from nine is 4; 3 from nine is 6; 0 from nine is 9; &c.——with ſuch expedition the Figure of the Ratio is found;

 and

and it's place is known by putting it down immediately over it's complement. As,

.246903
.7
. .5
. . .3
. . . .0
. 9
. 6

The Line of Ratios being formed by placing all the Ratios in the Approximation, as they come to hand, in one line like an entire Number reduces the Classical Numbers, as 1.2; 1.04; 1.006; 1.0009; 1.000003; &c. into 1.246903 a more concise expression, and of great use in placing the Logarithms. For as this *Line of Ratios* stands on the top of the Approximation, so it must also stand again on the top of the Logarithms, and then each Logarithm begins directly underneath it's respective Ratio, and continues towards the Right Hand. Each Figure and Place, in the *Line of Ratios*, refer to the same Figure and Class, numbered on the RADIX.

Note 1. Every Number before the Approximation begins must be put down as a root of 10. Which is done by placing the decimal point between the first figure and the second; For all Numbers between 1. and 10. are roots of 10.

Whole Number	8542.	put	8.542 Given root.
Mixt Number	63.87	put	6.387 Given root.
Decimal	.00013922	put	1.3922 Given root.

The form of Approximation will now be the same in all; only the Index, when the Logarithm is made, must be according to what the Given Number was at first; viz. + 3. + 1. and — 4 respectively; shewing how many places, above or below Units place, the highest figures stood.

Note 2. I give this as a General Principle; viz. That the Effect will be the same whether we Multiply a Given root Up to 10. 9. 8. 7. 6. 5. 4. 3. or 2. or to any other Number whose Logarithm is known: Or,

Or, instead of that, we first divide said Given Root by 10. 9. 8. 7. 6. 5. 4. 3. &c. respectively, and then multiply the quotient Up to 1.

This way of dividing the Given Root to bring it under 1. reduces all the various kinds of Ascending Approximations into one Rule, and that the best upon all accounts by far; chiefly because each figure in the *Line of Ratios* will stand in a perpendicular direction over it's complement from which it comes; also because the Ratio is obtained by subtracting it's complement from 9; and also because the succeeding part of the work is Multiplication only.

Note 3. The Divisor has always an Affirmative Logarithm, and the Multipliers have Negative Logarithms; by the *Reflected Rule* which changes the Signs.

Note 4. The First Divisor, or that of the First Approximation Up to 1. is the next Whole Number above the Given Root. As,

Given Root	1.0625	First Divisor	2.
Given Root	5.6391	First Divisor	6.
Given Root	9.8743	First Divisor	10.

Every New Divisor exhibits a New Class of Logarithms, which construct the Logarithm required, another new way.

EXAMPLE I.

To construct the Logarithm of Number 97. by dividing the Given root 9.7 by the next ascending Whole Number 10. and multiplying the Quotient 0.97 by Classical Numbers, Up to 1.

	1.03090080	Line of Ratios.	1.03090080	
Numb.	9.7		— 1283722	
	0.97		— 39069	
	0.9991		— 35	
	0.99999919		0.98677174	Log.

Here, to multiply the divided root .97 up to 1. I say 7 from nine is 2; but because 2 does not raise it's complement to 9. I put .03 Ratio; and adding Three-times 0.97 (two places lower) to itself, it

produces

produces .9991; Again I say, 1 from nine is 8; this being too little I put 9; viz. .0009 Ratio; and add Nine-times .9991, four places lower, to itself; which produces .99999919; Thus having approximated nines half way, I put down the Arithmetical Complement of the last four figures 9919; viz. 0080 into the Line of Ratios to complete the same.

The Line of Ratios being set down again, as above; I look out the Logarithms in the RADIX belonging to each Ratio, (viz. the Logarithm of 3 in the second class; the Logarithm of 9 in the fourth class; and the Logarithm of 8 in the seventh class) and having set them down, beginning with each Logarithm underneath it's respective Ratio, and mark'd them Negatives; I add them up together, but instead of setting down the *Sum*, subtract it as I go from the * Logarithm of Ten the Divisor, and set down the *Remainder*, which is the Logarithm of 9.7, and putting an Unit for the Index of the power, it will become the Logarithm of 97. viz. 1.98677174.

To construct the Logarithm of 97. a Second way; by dividing the Given root 9.7 by 100. and multiplying the Quotient 0.097 Up to 1.

	9.14128934	Line of Ratios.	9.14128934
Numb.	9.7		— 0.95424251
	0.097		— 4139269
	0.873		— 1703334
	0.9603		— 43408
	0.998712		— 8685
	0.99971071		— 3474
	0.99991065		— 391
			— 13
			— 2
			0.98677173 Log.

Here, I multiply the divided root .097 by 9. (to accelerate the approximation) which produces .873; Then I say, 8 from nine is 1 (viz. 0.1) and add Once .873 one place lower to itself, which produces .9603; Then 6 from nine is 3, but because 3 does not raise it's com-

* 1,00000000.

plement:

plement up to 9, I put in 4 (viz. .04) and add Four-times .9603 two places lower to itſelf, which produces .998712; Then 8 from nine is 1, (viz. .001) here I add Once .998712 three places lower to itſelf (rejecting what falls below eight places) and it produces .99971071|2; Then 7 from nine is 2 (viz. .0002 Ratio) here I add Twice .99971071 four places lower to itſelf (rejecting as aforeſaid) and it produces .99991065, which now being nines half way, the Arithmetical Complement of the other half 1065 that is 8934 fills up the Line of Ratios (viz. 9.14128934) as above.

Thus changing the Diviſor from 10. to 100. exhibits a New change of the Logarithms, which ſubtracted from * 2, the Logarithm of 100, produces the Logarithm of 97. another way.

The Third way is to divide the Given root by 1000, and to ſubtract the Negative Logarithms from 3, the Logarithm of 1000.

The Fourth way is to divide the Given root by 10000, and to ſubtract the Negative Logarithms from 4, the Logarithm of 10000. &c.

EXAMPLE II.

To conſtruct the Logarithm of Number 1111. by dividing the Given root 1.111 by the next aſcending Integer 2, and multiplying the Quotient up to 1.

	1.80010000	Line of Ratios.	1.80010000
2)	1.111		+ 0.30103000
	0.5555		— 25527251
	0.99990		— 4343
			0.04571406 Log.

To multiply the divided root 0.5555 up to 1. I add Eight-times .5555 one place lower to itſelf, which produces .99990, which being nines half way, the complement of the laſt four places 0000 is 9999, or ſhorter 10000 to complete the Line of Ratios. Thus the Logarithms of 8 in the Firſt Claſs, and of 1 in the Fourth Claſs ſubtracted from the + Logarithm of 2 the Diviſor, leaves the Logarithm of the Given root 1.111 as above; and becauſe the Number 1111. has three Figures above Units place; therefore it's Logarithm is 3.04571406.

* 200000000.

To

To construct the Logarithm of Number 1111. a Second way, by dividing the Given root 1.111 by 3. and multiplying the Quotient 0.37033333 up to 1.

	2.33831340	Line of Ratios.	2.33831340	
3)	1.111		+ 0.47712125	
	0.37033333		— .30103000	
	0.74066666		— .11394335	
	0.96286666		— 1283722	
	0.99175266		— 346053	
	0.99968668		— 13026	
	0.99998659		— 434	
			— 130	
			— 17	
			0.04571408	Log.

Here I divide the Given root by 3. and subtract the Negative Logarithms from the Logarithm of 3.

The Third way is to divide the Given root by 4, and subtract the Logarithms of the Multipliers from the Logarithm of 4.

The Fourth way is to divide the Given root by 5, and subtract the Logarithms of the Multipliers from the Logarithm of 5. &c.

EXAMPLE III.

To construct the Logarithm of Number 39784. by dividing the Given root 3.9784 by the next ascending Integer 4. and multiplying the Quotient up to 1.

	1.00542716	Line of Ratios.	1.00542716	
4)	3.9784		+ 0.60205999	
	0.9946		— 216606	
	0.9995730		— 17368	
	0.99997283		— 869	
			— 304	
			— 4	
			— 3	
			0.59970845	Log.

To

To multiply up the Quotient .9946; I say 4 from nine is 5, and Five-times .9946 added three places lower to itself produces .9995730; again 5 from 9 is 4; and Four-times .9995730 added four places lower to itself produces .99997282|92 half way nines; then the Arithmetical Complement of the other half 7283; viz. 2716 completes the Line of Ratios. Here the Logarithms of the Multipliers, subtracted from the Logarithm of the Divisor, leaves the Logarithm of the Given root 3.9784 as above, but if 4. be added it becomes the Logarithm of the Given Number 39784. viz. Logarithm 4.59970845, because 39784. has four figures above Units place.

To construct the Logarithm of Number 39784. a Second way, by dividing the Given root 3.9784 by 5, and multiplying the Quotient up to 1.

	1.24704029	Line of Ratios.		1.24704029
5)	3.9784		+	0.69897000
	0.79568			— 7918125
	0.954816			— 1703334
	0.99300864			— 302947
	0.99995970			— 1737
				— 9
				— 4
				0.59970844 Log.

Here I divide the Given root by 5, and subtract the Logarithms of the Multipliers from the Logarithm of 5.

The Third way is to divide by 6. and subtract the Logarithms of the Multipliers from the Logarithm of 6.

The Fourth way is to divide by 7, and subtract the Logarithms of the Multipliers from the Logarithm of 7. &c.

Example IV.

To construct the Logarithm of Number 537106492837. by dividing the Given root 5.37106492837 by the next ascending Integer 6. and multiplying the Quotient up to 1.

1.115485300707

	1.115485300707	Ratios.		1.115485300707
6)	5.371064928370		+	.778151250384
	0.895177488062		—	41392685158
	0.984695236868		—	4321373783
	0.994542189237		—	2166061757
	0.999514900183		—	173683058
	0.999914706143		—	34742169
	0.999994699319		—	2171467
	0.999999699292		—	130288
			—	304
			—	3
				0.730060402397 Log.

Here I divide by 6. and ſubtract the Logarithms of the Multipliers from the Logarithm of 6. which produces the Logarithm of the Given root as above; but 11.730060402397 is the Logarithm of the Given Number, becauſe it has 11 figures above Units place.

To conſtruct the Logarithm of Number 537106492837. a Second way, by dividing the Given root by 7. and multiplying the Quotient up to 1.

	1.285613990052	Ratios.		1.285613990052
7)	5.371064928370		+	.845098040014
	0.767294989767		—	79181246048
	0.920753987720		—	33423755487
	0.994414306738		—	2166061757
	0.999386378272		—	260498547
	0.999986010099		—	4342923
	0.999996009959		—	1302881
	0 999999009947		—	390865
			—	39087
			—	22
			—	1
				0.730060402396 Log.

Here I divide by 7. and ſubtract the Logarithms of the Multipliers from the Logarithm of 7.

The

The Third way is to divide by 8. and ſubtract the Logarithms of the Multipliers from the Logarithm of 8.

The Fourth way is to divide by 9. and ſubtract the Logarithms of the Multipliers from the Logarithm of 9. &c.

EXAMPLE V.

To conſtruct the Logarithm of Number 88972340154 66343. by dividing the Given root 8.897234015466343 by the next aſcending Integer 9. and multiplying the Quotient up to 1.

1.011534427910954 0	Ratios.	1.011534427910954 0
9) 8.8972340154663430	+	.9542425094393249
0.9885815572740381		— 43213737826426
0.9984673728467785		— 4340774793186
0,9994658402196253		— 2170929722302
0.9999655731397351		— 130286390285
0.9999955721069293		— 17371744533
0.9999995720892177		— 1737177580
0.9999999720890465		— 86858896
0.9999999920890459		— 30400614
		— 3908650
		— 43429
		— 3909
		— 217
		— 17
		0.9492550135523205 Log.

To conſtruct this Logarithm a Second way, by dividing the Given Root by 10. and multiplying the Quotient up to 1.

1.12173258724286950	1.12173258724286950	
0.88972340154663430	— 41392685158225	
0.97869574170129770	— 8600171761917	
0.99826965653532370	— 434077479318	
0.99926792619185900	— 30389978481	
0.99996741374019330	— 13028639028	
0.99999741276260550	— 868588095	
0.99999941275743100	— 217147186	
0.99999991275713740	— 34743557	
0.99999999275713040	— 3040061	
	— 86858	
	— 17371	
	— 8686	
	— 3474	
	— 261	
	— 39	
	— 2	
	0.9492550135523204	Log.

The Third way is to divide by 100. and ſubtract the Logarithms of the Multipliers from the Logarithm of 100.

The Fourth way is to divide by 1000. and ſubtract the Logarithms of the Multipliers from the Logarithm of 1000. &c.

Example VI.

To make the Logarithm of 3.14159265358979323846, &c. by dividing by 4. and multiplying the Quotient up to 1.

	1.26097441756768967587		1.26097441756768967587
4)	3.14159265358979323846	+	0.60205999132796239043
	0.78539816339744830962		— 79181246047624827722
	0.94247779607693797154		— 25305865264770240858
	0.99902646384155424983		— 3906892499101310
	0.99992558765901164865		— 3039954976139869
	0.99999558245014777947		— 173717445326642
	0.99999958243247758006		— 17371775801775
	0.99999998243231055305		— 434294479732

0.999-

0.9999999924323103773 7	— 304006136268
0.9999999999432310324 40	— 21714724090
0.9999999999932310324 12	— 260576689 1
	— 304006137
	— 26057669
	— 3474356
	— 390865
	— 26058
	— 3040
	— 217
	— 35
	— 3
	0.4971498726941338543 6 Log.

To divide by 5 and multiply the Quotient up to 1, does not vary the Logarithms infinitely; and therefore I divide by 6, and multiply the Quotient up to 1.

	1.9051881639267236465 2	1.90518816392672364652
6)	3.14159265358979323846	+ 0.77815125038364363251
	0.52359877559829887308	— .27875360095282896154
	0.99483767363676785885	— 21660617565076762 3
	0.99981186200495169814	— 434272768626696 4
	0.99991184319115219331	— 3474216888403320
	0.99999183613860748549	— 347434195787671
	0.99999983607329659435	— 43429446018 85
	0.99999993607328020168	— 2605766813247
	0.99999999607327636608	— 130288344376
	0.99999999907327635430	— 39086503354
	0.99999999999732763534 7	— 868588964
		— 260576689
		— 30400614
		— 868589
		— 130288
		— 26058
		— 1737
		— 261
		— 22
		— 1
		0.49714987269413385434

 The

The Fourth way is to divide by 7, and ſubtract the Logarithms of the Multipliers from the Logarithm of 7.

The Fifth way is to divide by 8, and ſubtract the Logarithms of the Multipliers from the Logarithm of 8.

Having now by this approximation carried the Logarithm to twenty figures, I have thus far accompliſhed my firſt deſign in computing the Claſſes to twenty figures; and in the courſe of this Problem have pointed out Four ways of making each Logarithm by approximating to 1.

But ſometimes it may be convenient to approximate quite up to Number 1. in order to ſave much labour, as in the following

EXAMPLE VII.

To conſtruct the Logarithm of Number 999. the ſhorteſt way, to Twenty places of Figures.

1.001001000001000000000	1.001001000001000000000
0.999	.00043407747931864066 9
0.999999	0434294264756156
0.999999999999	0434294482
1.000000000000000000000	0.99956548822598230869 2

It is evident, that Once .999 added three places lower to itſelf produces .999999; and this again added ſix places lower to itſelf produces Twelve nines: Now the Arithmetical Complement of the Nine loweſt places; viz. 000000000 is 999999999, inſtead of which I put 1000000000, which ſaves nine Logarithms by putting down one. But Once .999999999999 added twelve places lower to itſelf produces .999999999999999999999|999 or 1.0000000000, &c. Hence here are but three Ratios, and three Logarithms. This is the beſt way when the Given Number conſiſts of nines only.

In Example 1. I ſhew how to correct the Ratio when it was too little, by working with the Arithmetical Complement plus 1. But it will be equally true, if we follow the Arithmetical Complement altogether as the Ratio.

EXAMPLE

EXAMPLE VIII.

To conſtruct the Logarithm of Number 97, over again, without correcting the *Line of Ratios.*

	1		1	
	1.02070648	Ratios.	1.02070648	
Numb.	0.97		— 860017	
	194		— 432137	
	0.9894		— 30390	
	9894		— 261	
	0.999294		— 17	
	6995058		— 3	
	0.99999351		0.98677175	Log.

To conſtruct the Logarithm of Number 97. over again the Second way, without correcting the *Line of Ratios.*

	1 1		1 1	
	9.13090081	Ratios.	9.13090081	
Numb.	0.097		— 0.95424251	
	0.873		— 4139269	
	873		— 1283722	
	0.9603		— 432137	
	28809		— 39069	
	0.989109		— 4343	
	989109		— 35	
	0.99900009		0.98677174	Log.
	899100081			
	0.99989919			
	99989919			
	0.99999918			

Theſe two conſtructions are the ſame as thoſe in Example 1. only there I correct the Ratio when too low (for as the Complement it never is too high) which cauſes an obliteration of it's product, a thing one

one would choose to avoid, when done on paper. But here, after one 9 is got in the decimal (as in this first construction): Or when I have multiplied once (as in this second construction) I then take the Arithmetical Complements for Ratios, as they come, which will never raise the Multiplication above Unity by approximating to it. This, with the method here used in setting down each product and casting up in two lines, explains the rule in the most facile and clear manner.

I shall here observe, that when two Ratios succeed in one Clafs, (the second of which is always 1) one or more ciphers will follow in the Line of Ratios; as in the two constructions before us; but this is accidental, and yet it is the most likely to produce nines; because when we take the Arithmetical Complement for the Ratio, the product cannot be *too much*; but it will always be greater, by two Ratios as a Rectangle, than by one Ratio as a *Sum*; and consequently, by rising higher will be more apt to fill the approximation with nines, and the Line of Ratios with ciphers.

COROLLARY.

The *Line of Ratios* and Class of Logarithms may be varied indefinite ways; so that there shall not come two alike; and yet the same Logarithm shall be constructed.

Reverse Rule.

Given Numb.	0.3346788	(1)		1.0000000
	× 9		—	.9542425
	0.3012109	(2)		.0457575
	× 9		—	.9542425
	0.2710898	(3)		.0915150
	× 9		—	.9542425
	0.2439808	(4)		.1372725
	× 9		—	.9542425
	0.2195827	(5)		.1830300
	× 9		—	.9542425
	0.1976244	(6)		.2287875
	&c.			&c.

Here

Here each Product is a Number to approximate from; and the Logarithm oppofite is that which the Logarithms of the Multipliers are to be fubtracted from.

	3.1060400	(2)	3 1060400
N.	0.3012109	+	0.0457575
	0.9036327	—	.4771213
	0.9939959	—	.0413927
	0.9999599	—	25980
		—	174
			0.5246281 Log.

	5.0119996	(6)	5.0119996
N.	0.1976244	+	0.2287875
	0.9881220	—	.6989700
	0.9980032	—	043214
	0.9990012	—	04341
	0.9999003	—	3907
		—	391
		—	39
		—	3
			0.5246280 Log.

This fhews that Indefinite ways may be found to conftruct the Logarithm of any Number, without going higher than Multiplication.

Direct Rule.

Given Numb.		3.346788		
	÷	7.	+	0.84509804
		0.4781126	(1)	0.84509804
	÷	7.	+	.84509804
		0.6830180	(2)	0.69019608
	÷	7.	+	.84509804
		0.9757400	(3)	0.53529412
	÷	7.	+	.84509804
		0.1393914	(4)	0.38039216
	÷	7.	+	.84509804
		0.1991306	(5)	0.22549020
	÷	7.	+	.84509804
		0.2844723	(6)	0.07058824
		&c.		&c.

Here each Quotient is a Number to appproximate from; and the Logarithm oppofite is a Logarithm to fubtract the Negative Logarithms

rithms of the Multipliers from; to produce the Logarithm of Number 3346788 indefinitely as before; viz.

	2.0455536	(1)	2.0455536
N.	0.4781126	+	0.8450980
	0.9562252	—	.3010300
	0.9944742	—	170333
	0.9994466	—	21661
	0.9999463	—	2171
		—	217
		—	13
		—	3
			0.5246282 Log.

	5.0043644	(5)	5.0043644
N.	0.1991306	+	0.2254902
	0.9956530	—	.6989700
	0.9996356	—	17337
	0.9999355	—	1303
		—	261
		—	17
		—	2
			0.5246282 Log.

Hence both the Direct and Reverse Rules shew indefinite ways of constructing the Logarithm of any Number by multiplying up to 1.

The Indefinite Multipliers or Divisors may be changed at pleasure; but the Products and Quotients must be kept up close to the decimal point, with the highest figure of the decimal in the first place: Also in adding and subtracting the Logarithms, the Indices are every where to be rejected.

Given, Number 3346.788 = 10^n; to find n.

9)	3346.788	0.9542425
8)	371.8653333	0.9030900
7)	46.4831667	0.8450980
7)	6.6404524	0.8450980
	0.9486361	3.5475285

1.0539446		1.0539446
0.9486361	+	3.5475285
0.9960679	—	.0211893
0.9990561	—	13009
0.9999553	—	3907
	—	174
	—	17
	—	3
		3.5246282 = n.

That

That is, when a Number is put down as a Power of 10, then the Conſtruction brings out the whole Logarithm; viz. The Decimal and Characteriſtic together.

PROBLEM IV.

To find the Number correſpondent to any Logarithm, to Twenty places of Figures; by the *Reflected Rule*, approximating from the Given Logarithm to a Logarithm in the RADIX.

Here Negative Logarithms have numeral Multipliers, and Affirmative Logarithms numeral Diviſors.

The *Line of Ratios* ſtands diſtinguiſhed for it's Uſes in all Conſtructions, both of Logarithms and Numbers: When a Number is given it opens the RADIX for the Logarithm; and when a Logarithm is given it produces the Number by the continual Multiplication of it's parts.

Note. Every Logarithm, before the work begins, muſt be put down as the Logarithm of a Root of 10. By placing 0 for the Characteriſtic in the Units place. Thus,

Log. 4.827262 put 0.827262
Log. $\bar{3}$.123456 put 0.123456
Log. 0.029318 put 0.029318

Hence the Number produced will always be a root of 10. which is to be raiſed or depreſſed according to the given Index: As if the Index was + 4. then the decimal point in the Number muſt be moved four places more to the Right hand: But if the Index in the given Logarithm be — 3. the decimal point in the Number found muſt then be moved three places more to the Left hand.

The Rule I have ſelected as ſuperiour to all others in producing the Numbers from Logarithms, by the *Line of Ratios*, is, to ſubtract down to the Logarithm of 1. That is, to ſubtract claſſic Logarithms from the Logarithm given, till it is diminiſhed to 0.

EXAMPLE I.

Find the Number belonging to the Logarithm 1.8260748, or, 0.8260748.

	6.1151000	Line of Ratios.	6.1151000
Log.	0.8260748		1.0001000
	— .7781513		1.0051005
	479235		1.0151515
	— 413927		1.1166667
	65308		6.7000002 Numb.
	— 43214		
	22094		
	— 21661		
	433		
	— 434		

Note 2. As in making Logarithms, the highest figure of each Logarithm stands underneath it's Ratio; so now mutually, each Ratio must be placed over the highest figure of it's respective Logarithm.

Note 3. If any classic Logarithm be divided by Four, the first Quotient Figure shews the Quantity of it's Ratio: Therefore, after subtracting, I take the Fourth of the Remainder, The first Quotient Figure is the Ratio; and if not, it guides to the Class where it is.

Here I put down the Given Logarithm 1.8260748 as the Logarithm of a Root of Ten; viz. 0.8260748 (by Note 1.) I then look into the RADIX for the Logarithm next less than .8260, &c. which is .778151, &c. viz. The Logarithm of 6. Therefore I put 6. in the Units place of the *Line of Ratios*, and it's Logarithm underneath the given Logarithm; and having subtracted, the Remainder is 479235: By Note 3. I say, the Fours in 479235 is 1, &c. this first Quotient Figure I put as a Ratio in the first class, by Note 2. and at the same time put down it's Logarithm 413927, which subtracted leaves 65308: The Fours in 65308 is 1, &c. which first Figure (by Notes 2. 3.) I put as a Ratio in the second class, whose Logarithm 43214 being subtracted leaves 22094: The Fours in 22094 are 5, &c. which first Figure (by N. 2. 3.) I put as a Ratio in the third class, whose Logarithm

rithm 21661 being ſubtracted leaves 433: Laſtly, The Fours in 433 is 1, this firſt Figure as a Ratio I put in the fourth claſs, whoſe Logarithm 434 compared with 433, being nearly the ſame, terminates the Operation.

This dividing the Logarithm by Four, and taking the Firſt Quotient Figure for the Ratio, is an excellent way of finding it, and it's Logarithm by it: much better than to find the claſſic Logarithm firſt, and the Ratio by the Logarithm.

The *Line of Ratios* being now completed; the continual Multiplication of it's conſtituent parts, any way ordered, will produce the Number.

But becauſe the lower half of the *Ratios* as they ſtand in the *Line*, are always equal to the Product of their continual Multiplication; therefore I make the lower half of the *Line of Ratios* connected with Unity the Multiplicand, and the upper half, with the Integers, continual Multipliers. Thus, in finding Numbers, as before in making Logarithms, half the Short Multiplication is abſcinded.

Here having removed the *Line of Ratios* 6.1151000 as above; I put down underneath it, the lower half; viz. 1000, which connected with Unity becomes 1.0001000 the Multiplicand; conſequently 6.115 are the continual Multipliers.

Beginning with the loweſt, I add Five-times 1.0001000, three places lower to itſelf, which produces 1.0051005: Secondly, I add once 1.0051005 two places lower to itſelf, which produces 1.0151515: Thirdly, I add once 1.0151515, one place lower to itſelf, which produces 1.1166667: Laſtly, I multiply 1.1166667 by 6. and it produces the Number 6.7000002 a root of Ten; but moving the point one place forward (by Note 1.) makes it a Whole Number; viz. 67.00000 or 67. which was the Number required.

To produce the ſame Number a Second way, by adding to the Logarithm of the Root; viz. to 0.8260748 the Logarithm of 10; and dividing the produced Root of Ten, by 10.

	7.		7.	
	9.0632935	The Line of Ratios.	9.0632935	
Log.	1.8260748		1.0000935	
	— .9542425		1.0002935	
	8718323		1.0032944	
	— 8450980		1.0634921	
	267343		7.4444447	
	— 253059		6.7000002	Numb.
	14284			
	— 13009			
	1275			
	— 869			
	406			
	— 391			
	15			
	— 13			
	2			

Here I put down 1.8260748 which is the Logarithm of the required Root of Ten *plus* the Logarithm of 10. First I look in the RADIX for the next less Logarithm than the Position, which being that of the Integer 9. viz. .9542425; I put down 9. in the *Line of Ratios*, whose Logarithm also being put down and subtracted, the Remainder is .8718323: Secondly, I take out the Logarithm next less than this Remainder, which being that of the Integer 7. I put 7. (over the 9.) whose Logarithm being subtracted leaves 267343: Thirdly, The Fours in 26, &c. are 6; a Ratio of the second class (N. 2. 3.) whose Logarithm being subtracted leaves 14284: The Fours in 14, &c. = 3, I put in the third place (N. 2. 3.) whose Logarithm subtracted leaves 1275: The Fours in 12, &c. = 3, but because it's Logarithm is too much, I put 2 in the fourth place, whose Logarithm subtracted leaves 406: The Fours in 4, &c. is 1 in the fourth place, but because it's Logarithm is too much, I put 9 in the fifth place, whose Logarithm subtracted leaves 15: The Fours in 15 is 3 in the sixth place, whose Logarithm subtracted leaves 2. Lastly, The Fours in 20 is 5 in the seventh place, which completes the *Line of Ratios*.

From this *Line of Ratios*, the Number may be produced thus: I make the lower half (rather less than more) 935 connected with Unity; viz.

7
viz. 1.0000935 the Multiplicand, and the upper part; viz. 9.0632 continual Multipliers.

First I add Twice 1.0000935 four places lower to itself; which produces 1.0002935: Secondly, I add Three-times 1.0002935 three places lower to itself, which produces 1.0032944: Thirdly, I add Six-times 1.0032944 two places lower to itself, which produces 1.0634921: Fourthly, I multiply by 7. and it produces 7.4444447: Lastly, I multiply by 9. casting off the first figure of the product, and it produces the root 6.7000002: whence the Number 67. as before.

The Third way is, to add the Logarithm of 100, and to divide the produced Root by 100.

The Fourth way is, to add the Logarithm of 1000. and to divide the produced Root by 1000. &c.

Example II.

Given, the Logarithm 2.8790958795, to find the Number.

	7.0813224220	Line of Ratios.	7.0813224220	
Log.	0.8790958795		1.0000024220	
—	.8450980400		1.0000224220	
	339978395		1.0003224287	
—	334237555		1.0013227511	
	5740840		1.0814285712	
—	04340775		7.5699999984	Numb.
	1400065			
—	1302688			
	·97377			
—	86858			
	10519			
—	8686			
	1833			
—	1737			
	96			
—	87			
	9			
—	9			
	0			

Here

Here having put down the Logarithm of the Root; viz. 0.8790958795 I take from the RADIX the Logarithm next less; which is the Logarithm of 7. therefore I put 7. in the Units place of the *Line of Ratios*; whose Logarithm subtracted leaves 339978395: Now, the Fours in 33 is 8, a Ratio of the second place (because it must stand over the first figure of it's Logarithm by Note 2.) whose Logarithm subtracted leaves 5740840: The Fours in 5 is 1 in the third place (because it should stand over the 0 before it's Logarithm which begins with a cipher). See the RADIX in the next Problem, &c. &c.

In order to produce the Number, I remove the *Line of Ratios*, and put down the lower half connected with Unity; viz. 1.0000024220 under it for the Multiplicand; then the rest 7.08132 are continual Multipliers.

First, I add Twice 1.0000024220 five places lower to itself; which produces 1.0000224220: Secondly, I add Three-times 1.0000224220 four places lower to itself, which produces 1.0003224287: Once this added three places lower to itself produces 1.0013227511: Eight-times this added two places lower to itself produces 1.0814285712; which multiplied by 7. produces the Root 7.569999998, &c. consequently, moving the point two places forward (Note 1.) 757 is the Whole Number.

The reason why the Ratios 1 and 2 only, are every where put a place higher than the first figure of their respective Logarithms, is evident, if we consider that the first place in both is always 0. for the Logarithm is less than half the Ratio.

To produce the same Number a Second way; viz. by adding to the Logarithm of the Root; viz. to 0.8790958795 the Logarithm of 10. and dividing the produced Root of Ten, by 10.

Upon trial I soon find this Position does not vary the *Line of Ratios* infinitely; and therefore I pass on to the Third way; that is, To add the Logarithm of 100, and to divide the produced Root by 100. Thus,

	3		3
	9.0381625565	Line of Ratios.	9.0381625565
Log.	2.8790958795		1.0000025565
—	2.8627275283		1.0000625567
	163683512		1.0001625630
—	128372247		1.0081638635

35311265

35311265	1.0384087794
— 34605321	9.3456790146
705944	8.4111111111
— 434273	7.5700000000 Numb.
271671	
— 260569	
11102	
— 8686	
2416	
— 2171	
245	
— 217	
28	
— 26	
2	

Here the Logarithm of the Root increased by the Logarithm of 100. is 2.8790958795: Now because here is no Logarithm in the RADIX high enough to take off this Position; I try how many times the Logarithm of 9. is contained in it; which being three-times, I put $\overset{3}{9}$. in the *Line of Ratios*, and three-times the Logarithm of 9. underneath the increased Logarithm, which subtracted leaves 163683512: Secondly, The Fourth of 16 is 4; but its Logarithm being greater than 1636, &c. I put 3 in the Second place, &c. &c.

Having removed the *Line of Ratios*, I connect the lower half with Unity, making 1.0000025565 the Multiplicand, which I multiply continually by $\overset{3}{9}.03816$, to produce the root.

In this Operation there is nothing new or difficult, except what relates to $\overset{3}{9}$. which is thus; I multiply 1.0384087794 by 9 (units) and it produces 9.3456790146: this multiplied by 9 (tenths) produces 8.4111111131 corrected 8.4111111111: this multiplied again by 9 (tenths) produces the root 7.5700000000 true, whence the Number 757. as before.

Here .9 × .9 being $\frac{9}{10} \times \frac{9}{10}$ (casting off one figure each time) amounts to the same thing as dividing the root by 100.

The

The Fourth way is to add the Logarithm of 1000. and to divide the produced Root by 1.000. &c.

EXAMPLE III.

Given, the Logarithm 0.31808083647977 of fourteen figures, to find the Number, to fourteen figures, corresponding thereto.

	2.04004029953217	Ratios.	2.04004029953217	
Log.	0.31808083647977		1.00000009953217	
	— 30102999566398		1.00000029953219	
	1705084081579		1.00004029954417	
	— 1703333929878		1.04004191152594	
	1750151701		2.08008382305188	Numb.
	— 1737143185			
	13008516			
	— 8685889			
	4322627			
	— 3908650			
	413977			
	— 390865			
	23112			
	— 21715			
	1397			
	1303			
	94			
	— 87			
	7			
	— 4			
	3			

This Process is evidently the same, with regard to form, as in the foregoing Examples: But what I would here observe is, that in this Operation, the Cube Root of 9. is extracted to fourteen places: For, the Logarithm of 9. which stands in the RADIX, is 0.95424250943932, &c. the $\frac{1}{3}$ of which is the given Logarithm 0.31808083647977, &c. whose

whose Number is here found to be 2.080083823051 8, &c. the Cube Root of 9.

To produce the same Number a Second way, by adding to Logarithm 0.31808083647977 the Logarithm of 10; and dividing the produced Root of Ten, by 10.

	2.		2.	
	9.15052130872840	Ratios.	9.15052130872840	
			1.00000000872840	
Log.	1.31808083647977		1.00000030872840	
	— .95424250943932		1.00000130872871	
	36383832704045		1.00002130875488	
	— 30102999566398		1.00052131940926	
	6280833137647		1.05054738537972	
	— 4139268515823		1.15560212391769	
	2141564621824		2.31120424783538	
	— 2118929906994		2.08008382305184	Numb.
	22634714830			
	— 21709297223			
	925417607			
	— 868580278			
	56837329			
	— 43429426			
	13407903			
	— 13028833			
	379070			
	— 347436			
	31634			
	— 30401			
	1233			
	— 869			
	364			
	— 347			
	17			
	— 17			
	0			

In multiplying 2.31120424, &c. by 9 (tenths) I cast off one figure, which divides the produced Root by 10. This Cube Root is the same as by the First Way, to fourteen figures.

The Third way is to add the Logarithm of 100; and to divide the produced Root by 100.

 The

The Fourth way is to add the Logarithm of 1000; and to divide the produced Root by 1000, &c.

EXAMPLE IV.

Given, Logarithm 0.000058052874164214 of eighteen places, to find the Number to eighteen places correspondent thereto.

	1.000133677136996137	Ratios.	1.000133677136996137	
Log.	0.000058052874164214		1.000000000136996137	
	— 43427276862670		1.000000007136996138	
	14625597301544		1.000000077136996638	
	— 13028639028489		1.000000677137042920	
	1596958273055		1.000003677139074331	
	— 1302881491388		1.000033677249388503	
	294076781667		1.000133680617113442	Numb.
	— 260576610969			
	33500170698			
	— 30400612669			
	3099558029			
	— 3040061363			
	59496666			
	— 43429448			
	16067218			
	— 13028834			
	3038384			
	— 2605767			
	432617			
	— 390865			
	41752			
	— 39087			
	2665			
	— 2606			
	59			
	— 43			
	16			
	— 13			
	3			

In this Operation the 365th Root of 1.05 is extracted to eighteen places; viz. $\overline{1.05}^{\frac{1}{365}} = 1.00013368061711344$, &c. For the Logarithm of 1.05 (or

(or five in the second class); viz. Logarithm 0.021189299069938073, &c. divided by 365, quotes the given Logarithm, &c. &c.

To produce the same Number the Second way, by adding to the Logarithm 0.000058052874164214 the Logarithm of 10; and dividing the produced Root of Ten, by 10.

	9.110233697134998139	Ratios.	9.110233697134998139
Log.	1.000058052874164214		1.000000000134998139
	— 954242509439324875		1.000000007134998140
	45815543434839339		1.000000097134998782
	— 41392685158225041		1.000000697135057063
	4422858276614298		1.000003697137148468
	— 4321373782642574		1.000033697248062582
	101484493971724		1.000233703987512195
	— 86850211648957		1.010236041027387317
	14634282322767		1.111259645130126049
	— 13028639028489	Numb.	1.000133680617113444
	1605643294278		
	— 1302881491388		
	302761802890		
	— 260576610969		
	42185191921		
	— 39086501612		
	3098690309		
	— 3040061363		
	58628946		
	— 43429448		
	15199498		
	— 13028834		
	2170664		
	— 1737178		
	433486		
	— 390865		
	42621		
	— 39087		
	3534		
	— 3474		
	60		
	— 43		
	17		
	— 13		
	4		

Here

Here multiplying the last time by Nine; I cast off one figure from the product, which is the same as multiplying by 9 (tenths) or dividing the produced Root of Ten by 10. By this Second way the Root of 1.05 is the same as before to eighteen figures; exhibiting the Compound Interest of 1 £. for one day in the Decimal; and the amount of 1 £. for one day in the Whole, at 5 £. per cent. per annum.

The Third way is to add the Logarithm of 100; and to divide the produced Root by 100.

The Fourth way is to add the Logarithm of 1000; and to divide the produced Root of Ten, by 1000, &c.

In the RADIX the Second Class has these Numbers; viz. 1.09 1.08 1.07 1.06 1.05 1.04 1.03 1.02 and 1.01; which are the Annual Amounts of 1 £. at Nine, Eight, Seven, Six, Five, Four, Three, Two and One Pound per cent. respectively; Therefore if the Logarithm of any one of these, taken out of the same Class, be divided by 365 to Twenty Figures in the Quotient; a Process like this will exhibit the daily Amount at that rate of Compound Interest, to Twenty Figures in the Number.

EXAMPLE V.

Given, the Logarithm 0.0000000001; to find the Number.

1.00000000002302585093197

L. 0.0000000001 . . . .

— 868588963720

131411036280

— 130288344569

1122691711

— 868588964

254102747

— 217147241

36955506

— 34743559

2211947

— 2171472

40475

— 39087

1388

— 1303

85

— 43

42

— 39

3

Subtraction down to o.

1.00000000002302585093197

1.00000000000302585093197

1.00000000002302585093258 N.

* 230258509.

The Second, Third, Fourth, &c. ways might be applied here as uſual; but then they would be much more operoſe by the addition of the Indices, as is evident: Therefore a Second proof neareſt this (in conciſeneſs) which ſubtracts down to the Logarithm of 1; is another Principle, that adds up to the Logarithm of 1; viz.

Second Way.

1.00000000000697414906575	1.00000000000697414906575
— 0.00000000001302883445514	0.9999999999902585093425
L. 0.0000000001	1.00000000002902585093396
+ 260576689134	1.00000000002302585093258 N.
1260576689134	
+ 39086503371	
1299063192505	
+ 3040061373	
1302703253878	
+ 173717793	
1302876971071	
+ 4342945	
1302881314616	
+ 1737178	
1302883051794	
+ 390865	
1302883442659	
+ 2606	
1302883445265	
+ 217	
1302883445482	
+ 30	
1302883445512	
+ 2	
1302883445514	

Addition up to 0.

* 230258509.

Here I add continually to the given Logarithm ſuch affirmative Logarithms as will make it ballance the negative Subſtitute, ſo that the whole

whole Sum of the Logarithms when cast up together may be equal to the Logarithm of 1.

Having thus completed the *Line of Ratios*, when removed I subtract the lower half 97414906575 from Unity (whose Logarithm this Rule adds up to); Then I multiply the Remainder by 1.0000000003 whose Logarithm is —; and divide by the upper half 1.00000000006 whose Logarithm is +; and it produces the Number the same as before.

In this Example the 10000000000th Root of 10. is extracted two ways, wherein the figures * 230258509 are an approximation of Lord *Neper*'s Hyperbolic Logarithm of 10. which expression 1.000000000, &c. 230258509, &c. each infinite, gives some Idea of the infinite Root of 10.

EXAMPLE VI.

To find the Number corresponding to Logarithm 0.61661644 indefinite ways, by Addition of Indices and Subtraction down to 0.

First Way.

	4.03396357	Ratios.	4.03396357	
Log.	0.61661644		1.00006357	
	— .60205999		1.00096363	
	1455645		1.00396652	
	— 1283722		1.03408552	
	171923		4.13634208	Numb.
	— 130093			
	41830			
	— 39069			
	2761			
	— 2606			
	155			
	— 130			
	25			
	— 22			
	3			

Second

Second Way.

From the given Logarithm + the Logarithm of 10, subtract the Logarithm of 9, &c.

	4.14435512	4.14435512	
Log.	0.61661644	1.00005512	
—	.95424251	1.00035514	
	.66237393	1.00435656	
—	.60205999	1.04453082	
	6031394	1.14898390	
—	4139269	4.59593560	
	1892125	4.13634204	Numb.
—	1703334		
	188791		
—	173371		
	15420		
—	13027		
	2393		
—	2171		
	222		
—	217		
	5		
—	4		
	1		

Third Way.

From the First Remainder .66237393 + the Logarithm of 10, subtract the Logarithm of 9, &c.

	5.02129285	Ratios.	5.02129285
Log.	0.66237393		1.00009285
—	.95424251		1.00029287

.70813142

.70813142	1.00129316	
— .69897000	1.02131902	
916142	5.10659510	
— 860017	4.59593559	Similar line.
56125		
— 43408		
12717		
— 8685		
4032		
— 3908		
124		
— 87		
37		
— 35		
2		

Fourth Way.

From the First Remainder .70813142 + the Logarithm of 10, subtract the Logarithm of 9. &c.

	5.13158712	Ratios.	5.13158712	
Log.	0.70813142		1.00008712	
	— .95424251		1.00058716	
	.75388891		1.00158775	
	— 69897000		1.03163538	
	5491891		1.13479892	
	— 4139269		5.67399460	
	1352622		5.10659514	Similar line.
	— 1283722			
	68900			
	— 43408			
	25492			
	— 21709			
	3783			
	— 3474			
	309			
	— 304			
	5			
	— 4			
	1			

Fifth

Fifth Way.

From the First Remainder .75388891 + the Logarithm of 10, subtract the Logarithm of 9. &c.

	6.05070449	6.05070449
Log.	0.75388891	1.00000449
	— .95424251	1.00070449
	.79964640	1.05073971
	— .77815125	6.30443826
	2149515	5.67399443 Similar line.
	— 2118930	
	30585	
	— 30390	
	195	
	— 174	
	21	
	— 17	
	4	
	— 4	
	0	

Sixth Way.

From the First Remainder .79964640 + the Logarithm of 10, subtract the Logarithm of 9, &c.

	7.00070449	7.00070449
Log.	0.79964640	1.00000449
	— .95424251	1.00070449
	.84540389	7.00493143
	— 84509804	6.30443829 Similar line.
	30585	
	— 30390	
	195	
	— 174	
	21	
	— 17	
	4	

Here

Here the Transition is made by the First Remainder, which is a continuation of the given Logarithm + the Logarithm of 10, — the Logarithm of 9. &c. rejecting the Indices; and is the same in effect as adding (the Indices 1. 2. 3. 4. 5. &c. i. e.) once, twice, three times, &c. the Logarithm of 10, and subtracting an equimultiple of the Logarithm of 9.

In producing the Number I multiply the lower half connected with Unity, by the upper half of the *Line of Ratios*, thus far as usual: But in multiplying by 9 (tenths) I cast off at each time One Figure; and continue till One Line falls *Similar*.

This *Line*, by reviewing the preceding, you will find to be *Similar* to a line there; and as far as these lines agree, just so far will the Numbers agree: Therefore by terminating every numeral Process at the first *Similar Line*, they are rendered equally concise, and the Proof is as certain as if multiplied by 9 (tenths) down to the Number. Here .61661644 is the Logarithm Sine, and 4136342 the natural Sine of 24°: 26'.

PROBLEM V.

To illustrate the Use of LOGARITHMS by the CLASSICAL NUMBERS.

The Rules of ARITHMETIC wherein Logarithms may be employed are these; viz. Multiplication, Division, Involution and Evolution.

EXAMPLE I.

To multiply 9876543210 into 1234567890 by adding their Logarithms together.

9.9946049681	Log. of Multiplicand.
9.0915149772	Log. of Multiplier.
19.0861199453	Log. of Product.

The Sum of the two Logarithms, when brought into Numbers, by the preceding, will shew the Product.

	1.2160445440	1.2160445440	
Log.	0.0861199453	1.0000045440	
	— 791812460	1.0000445442	
	69386993	1.0060448115	
	— 43213738	1.0161052596	
	26173255	1.2193263115	Numb.
	— 25979807		
	193448		
	— 173714		
	19734		
	— 17372		
	2362		
	— 2171		
	191		
	— 174		
	17		
	— 17		
	0		

121932631100000000000. by Logarithms.
121932631112635269000. by Numbers.

Here the ten highest figures are the same in both, which proves that these Logarithms of ten figures, extend to ten figures, in the Multiplicand, Multiplier and Product.

Example II.

To divide 9876543210. by 1234567890. by subtracting the Logarithm of the Divisor from the Logarithm of the Dividend.

9.9946049681	Log. of Dividend.
9.0915149772	Log. of Divisor.
0.9030899909	Log. of Quotient.

The

The difference of the two Logarithms, when brought into Numbers, will exhibit the Quotient.

	8.0000000090		8.0000000090	
Log.	0.9030899909		1.0000000090	
	— .9030899870		8.0000000720	Numb.
	39			
	— 39			
	0			

8.000000072 by Logarithms.
8.000000072 by Numbers.

The Quotients here being the ſame to ten figures, proves that theſe Logarithms of ten figures, extend alſo to ten figures in the Diviſor, Dividend and Quotient.

Example III.

To involve *Number* 1.05 to the *Eight Hundredth Power* by multiplying it's Logarithm into 800, and reducing the Product into Numbers.

	8.11608572683856969 5	Ratios.	8.116085726838569695
L,	16.951439255950458235		1.000000000838569695
	— .903089986991943586		1.000000000683856970 0
	48349268958514649		1.000000026838569837
	— 41392685158225041		1.000000726838588624
	6956583800289608		1.000005726842222817
	— 4321373782642574		1.000085727300370195
	2635210017647034		1.006086241664172416
	— 2597980719908592		1.016147104080814140
	37229297738442		1.117761814488895554
	— 34742168884033		8.942094515911164432 N.
	2487128854409		
	— 2171466980853		

315661873556
— 304006030930
11655842626
— 8685889551
2969953075
— 2605766884
364186191
— 347435585
16750606
— 13028834
3721772
— 3474356
247416
— 217147
30269
— 26058
4211
— 3909
302
— 261
41
— 39
2

I multiply the Logarithm of 1.05 or 5 in the Second Class, by 800, which produces Logarithm 16.95143925595045823479200, here I cast off three places from the product as defective, and proceed with 16.95143925595045823479 as true: But the Index 16. shews that there will be 17 places of Figures in Whole Numbers; and therefore I take 18 Figures in the Logarithm as sufficient to produce them: Whence $1.05^{800} = 89420945159111644$ £.

Example IV.

To involve 1.04 to the $5\frac{80}{365}$ Power = the 5.21917808219, &c. Power; by multiplying it's Logarithm 0.0170333393 into the decimal Index; and finding the Number.

80

80
240
2160
0.21920
5.21917808219
3933330710.0 inverted Log. of 1.94.
521917808
365342466
1565753
156575
15658
1566
470
16
0.0889000312 Log. of the Power.

This short way of reducing $\frac{80}{365}$ (80 days) into decimals, is thus: I multiply 80 by .00274 or .00273; viz. I put down 80 and multiply it by 3 which produces 240; this multiplied by 9 produces 2160; the Sum of these is 0.21920 = 0.2192 = 0.2191999999, &c. The nines are not to be put down, but the Arithmetical Complement in their stead; viz. The Arithmetical Complement of .2191 annexed to it is 0.21917808, and the Arithmetical Complement of 7808 annexed to it makes 0.219178082191, &c. infinitely. Thus 5 Years 80 Days is 5.21917808219178O8, &c. Years. Having got the Logarithm of the Power, I reduce it into Numbers.

	1.2225778275	Ratios.	1.2225778275	
Log.	0.0889000312		1.0000078275	
	— 791812460		1.0000778280	
	97187852		1.0005778669	
	— 86001718		1.0025790226	
	11186134		1.0226306031	
	— 8677215		1.2271567237	Numb.
	2508919			
	— 2170930			

$$
\begin{array}{r}
337989 \\
-\ \underline{303995} \\
33994 \\
-\ \underline{30401} \\
3593 \\
-\ \underline{3474} \\
119 \\
-\ \underline{87} \\
32 \\
-\ \underline{30} \\
2
\end{array}
$$

This Number 1.2271567237, which is the Amount of 1 £. for 5 Years 80 Days; at 4 £. per cent. per Annum, Compound Interest; when multiplied into any principal Sum, will produce the Amount of that Sum, on the same terms: Thus, if the above Number be multiplied into 169 £. the Product 207.38948630 = £. 207 : 7 : 9 $\frac{1}{4}$ is the Amount of 169 £. for 5 Years, 80 Days, &c.

But this Example is intended as a general Rule for casting up Compound Interest for any Term of Years and Days (so that the whole Amount does not exceed 22 Figures) at any rate from 10 £. down to 1 £. per cent. And also for any other rate not found in the Classes, by first making a Logarithm for the rate proposed.

EXAMPLE V.

To extract the $\frac{287}{365}$ Root of 1.05 by finding the Number to it's Logarithm .0211892991 × $\frac{287}{365}$; viz.

		Ratios.	
	1.0388371215		1.0388371215
Log.	0.0166611749		1.0000071215
	— 128372247		1.0000371217
	38239502		1.0008371514
	— 34605321		1.0088438486
	3634181		1.0391091641 Numb.
	— 3472967		

$$
\begin{array}{r}
161214 \\
-\ 130286 \\
\hline
30928 \\
-\ 30401 \\
\hline
527 \\
-\ 434 \\
\hline
93 \\
-\ 87 \\
\hline
6 \\
-\ 4 \\
\hline
2
\end{array}
$$

This Number 1.039109164 exhibits the ſame Amount of 1 £. for 287 Days, as if done by the above general Rule, but,

Note, When the Product Minus 1, as $\overline{287 \times .00274} - 1 = 0.78637$ has Five Figures; the higheſt Figure muſt be ſubtracted alone, thus 0.78637—7 is 0.78630, then annex the Arithmetical Complement of the Four loweſt Figures repeatedly; viz. 0.7.8630.1369.8630.1369. &c. $= \frac{287}{365} =$ 287 Days.

Note 2. Number $16 \times 7 = 14 \times 8 = 112$; Alſo $999 \div 9 = 111$ (Ex. 7. Prob. 3.) therefore the addition and ſubtraction of the Logarithms, reſpectively, will make the Logarithms of 1.12 and 1.11, which are the Amounts of 1 £. at 12 and 11 per cent.

LOG-

Logarithmic Canon,

Of the yearly and daily Amounts of 1 £.

For computing Compound Interest the shortest way.

Rates.	Log. of the yearly Amounts.	Log. of the daily Amounts.
1.10	.0413926851582250407502	.0001134046168718494267 13
1.09	.0374264979406236352005	.0001025383505222565347 96
1.08	.0334237554869497023126	.0000915719328409580885 28
1.07	.0293837776852096408346	.0000805035005074236735 19
1.06	.0253058652647702408467	.0000693311377116992899 91
1.05	.0211892990699380727935	.0000580528741642138980 64
1.04	.0170333392987803548477	.0000466666830103571365 69
1.03	.0128372247051722051710	.0000351704786443074114 28
1.02	.0086001717619175610490	.0000235621144162124960 25
1.01	.0043213737826425742752	.0000118393802264180117 13
per £.	To be multiplied by Years.	To be multiplied by Days.

In this Canon the First Column exhibits the Rates per £. or per Cent. the Second Column shews their Logarithms as they stand in the Classes; and the Third Column which is every where the 365th part of the Second, shews the Logarithms of the daily Amounts of the Rates respectively.

To compute the Amount of 1 £. for 5 Years and 80 Days, at 4 £. per Cent. per Annum, Compound Interest, by this Table.

1,04 Log. .0170333393		Log. .000046666683	
× 5	Years.	× 80	Days.
.0851666965		.003733334640	

.0851666965
.0037333346
.0889000311

 The

The Aggregate Logarithm .0889000311 reduced into Numbers will shew the Amount of one Pound, as in Example 4.

EXAMPLE VI.

To extract the Sursolid ($\frac{1}{5}$) Root of the following Decimal Fraction; viz. 0.0006982544072895; by making it's Logarithm first, then dividing by five; and finding the Number.

	1.4228992194	1.4228992194
Numb.	0.6982544073	.1461280357
	0.9775561702	86001718
	0.9971072936	8677215
	0.9991015082	3472967
	0.9999007894	390847
	0.9999907805	39086
		869
		43
		39
		2
		—4.8440136857 Log.

Here I put — 4. for the Index, because the quantity of the Decimal Resolvend begins upon the fourth place below Units: Therefore the Logarithm — 4. +.8440136857 being heterogeneous, I subtract the decimal parts from the Index, and it becomes — 3.1559863143 homogeneous, or wholly negative; which now being divided by the Denominator of the Root; viz. 5 quotes 0.6311972628 negative; then subtracting the Decimal from the Index 0. it leaves $\bar{1}$.3688027372 the Logarithm of the Root restored to it's proper heterogeneous form for finding the Number; viz.

	2.1624754960	Ratios.	2.1624754960
Log.	0.3688027372		1.0000054960
	.3010299957		1.0000754964
	677727415		1.0004755266
	413926852		1.0024764777

263800563	1.0626250664
253058653	1.1688875730
10741910	2.3377751460 Numb.
8677215	
2064695	
1736831	
327864	
303995	
23869	
21715	
2154	
1737	
417	
391	
26	
26	
0	

But because the Index of the Logarithm of the Surſolid is — 1; therefore the firſt ſignificant figure of the Root is one place below Units, i. e. in the firſt place of Decimals; viz. 0.2337775146 the Surſolid Root required; which involved to the 5th power produces 0.000698254407; every figure of which is true.

If it was required to extract the ſecond Surſolid ($\frac{1}{7}$) Root of a large Whole Number, as 9876543210. whoſe Logarithm is 9.9946049682; Here the $\frac{1}{7}$ of 9.9946049682 is 1.4278007097; viz. The Logarithm of the Root, whoſe Index + 1. ſhews the firſt Figure in the Number is one place above Units, i. e. the ſecond in Whole Numbers.

This New Method of Involution and Extraction evidently excels any thing of the kind done by Natural Numbers; which if it takes place at the Cube or Surſolid Root, it is manifeſt the higher, and the more involved the Extraction is, the more the advantage will obtain in favour of this Rule.

EXAMPLE VII.

To construct Logarithmic Sines, Tangents, Secants, &c. from the Natural.

34° : 0′

	1.75184318	1.75184318
N. Sine.	0.5591929	.23044892
	0.95062793	2118930
	0.99815933	43408
	0.99915749	34730
	0.99995682	1737
		130
		4
		3
		0.74756166 Log. Sine.

26° : 54′

	1.93720728	Ratios.	1.93720728
N. Tang.	0.5073290		.27875360
	0.96392510		1283722
	0.99284285		302947
	0.99979275		8685
	0.99999271		304
			9
			3
			0.70528970 Log. Tang.

78° : 19′

	2.01248632	Ratios.	2.01248632
N. Sec.	0.49382120		.30103000
	0.98764240		432137
	0.99751882		86772
	0.99951386		17368
	0.99991367		3473
			260
			13
			9
			0.69356968 Log. Sec.

10° : 1′

N. Verf.

	1		1
	6.08239412	Ratios.	6.08239412
N. Versed	0.152428 . .		.77815125
	0.914568 . .		3342376
	0.98773344		432137
	0.99761077		86772
	0.99960599		13027
	0.99990587		3907
			174
			4
			1
			0.18306477 Log. Versed Sine.

COROLLARY.

Hence it appears, that not only the TABLE of Logarithms in *Sherwin* for Numbers; but also his TABLES of Logarithmic Sines, Tangents, Secants and Versed Sines, to every Minute of the Quadrant, are constructed from the Natural Numbers, Sines, Tangents, Secants, &c. each Ten different ways, and *vice versa* (Ex. 6. Prob. 4.) By a RADIX of Four Classes, containing only Forty-six Logarithms. P. 80.

CONSTRUCTION.

The Classical RADIX was constructed to Twenty-three places in Number and Logarithm, by two others constructed for that purpose; the one consisting of Square-Square Roots, and the other of Cube-Square Roots.

1. The Square-Square RADIX is made by extracting the Square Root of Ten to 22 or 23 places in Decimals, and then a continuation of Square Roots out of that Root, and to the same length, till the Resolvend 10 is extracted down to 1; and by halving the Logarithm of Ten as often as the Square Root of Ten is extracted; at which time the Logarithm 1, will be reduced to 0.

2. The Cube-Square RADIX is made by extracting one Cube Root of Ten, to 23 places in Decimals, and out of that Cube Root a continuation of Square Roots, extended to the same length, till the Resolvend

Resolvend 10. is extracted down to 1; And by thirding the Logarithm of Ten once for the Cube Root, to 23 places in decimals; and then halving afterwards as often as the Square Root is extracted; when the Logarithm 1, will thereby be reduced to 0.

These Two, being in subtriplicate proportion, are invested with a peculiar quality, which makes them superiour to any other First Constructions; viz. Their Numbers respectively will at certain intervals, by Multiplication and Division, produce the same Number; and their Logarithms, by Addition and Subtraction, produce the same Logarithm.

By this meeting of the Numbers and Logarithms, the Work is proved six, seven, eight, nine, or ten times, in the course of one Approximation. Therefore beginning always with the Cube-Square first, whose Logarithms run out alternately by 333333 infinite, or by 666666 infinite; when the Addition of these produce 999999, &c. or the Subtraction 000000, &c. I then stop here; and turn to the Square-Square RADIX, which in it's Approximation will produce the same thing.

It was upon two *Abacus's placed together, that I worked off the heavy Numeral Part of this Calculation; where (by making *Tariffas* of the Factors or Roots on paper) my Multiplication and Division were only Addition and Subtraction, which have no mental reckoning.

The Result of this large Calculation, every where attested by a Quadruple Proof, is the following RADIX of Twelve Classes, extended to Twenty-three Figures; Twenty-two of which are true.

Radix of Twelve Classes.

N. 10.		L. 1.
0	9	.95424250943932487459006
	8	.90308998699194358564121
	7	.84509804001425683071220
	6	.77815125038364363250877
	5	.69897000433601880478626
	4	.60205999132796239042748
	3	.47712125471966243729503
	2	.30102999566398119521374
	1	.00000000000000000000000

	1	9	.27875360095282896153632
		8	.25527250510330606980380
		7	.23044892137827392854018
		6	.20411998265592478085495
		5	.17609125905568124208129
		4	.14612803567823802592594
		3	.11394335230683676920649
		2	.07918124604762482772251
*		1	.04139268515822504075018
	2	9	.03742649794062363520049
		8	.03342375548694970231257
		7	02938377768520964083456
		6	.02530586526477024084673
		5	.02118929906993807279349
		4	.01703333929878035484770
		3	.01283722470517220517104
		2	.00860017176191756104895
		1	.00432137378264257427520
	3	9	.00389116623691052171524
		8	.00346053210950648615722
		7	.00302947055361800716934
		6	.00259798071990859231196
		5	.00216606175650767623042
		4	.00173371280900052976800
		3	.00130093302041811880084
		2	.00086772153122691249282
		1	.00043407747931864066887
	4	9	.00039068924991013102890
		8	.00034729668536354068771
		7	.00030389978481249181048
		6	.00026049854739034681783
		5	.00021709297223020828189
		4	.00017368305846491882266
		3	.00013026880522706100380
		2	.00008685021164895722887
		1	.00004342727686266963731

1	5	9	.0000390847445841673924З
		8	.00003474216888403320052
		7	.00003039954976139869403
		6	.00002605688721539547945
		5	.00002171418124515513695
		4	.00001737143184980922154
		3	.00001302863902848926075
		2	.00000868580278032675717
		1	.00000434292310445318686
	6	9	.000000390863274830828222
		8	.000000347434195787671284
		7	.000000304005073315761021
		6	.000000260575907415010580
		5	.000000217146698085333086
		4	.000000173717445326641696
		3	.000000130288149138849553
		2	.000000086858809521869793
		1	.000000043429426475615565
	7	9	.000000039086485782376700
		8	.000000034743544654844137
		7	.000000030400603093017780
		6	.000000026057661096897562
		5	.000000021714718666483375
		4	.000000017371775801775142
		3	.000000013028832502772776
		2	.000000008685888769476191
		1	.000000004342944601885293
	8	9	.000000003908650161240011
		8	.0000000034743555716251788
		7	.000000003040061266920620
		6	.000000002605766813246504
		5	.000000002171472355229447
		4	.0000000017371777892869448
		3	.000000001302883426166504
		2	.000000000868588955120614
		1	.000000000434294479731781

	9	9	.00000000390865031954035
		8	.00000000347435584132858
		7	.00000000304006136268255
		6	.00000000260576688360223
		5	.00000000217147240408756
		4	.00000000173717792413865
		3	.00000000130288344375544
		2	.00000000086858896293793
		1	.00000000043429448168608
	10	9	.00000000039086503353703
		8	.00000000034743558538362
		7	.00000000030400613722589
		6	.00000000026057668906377
		5	.00000000021714724089732
*		4	.00000000017371779272657
		3	.00000000013028834455142
		2	.00000000008685889637196
		1	.00000000004342944818817
	11	9	.00000000003908650336953
		8	.00000000003474355855087
		7	.00000000003040061373219
		6	.00000000002605766891341
		5	.00000000002171472409463
		4	.00000000001737177927580
		3	.00000000001302883445689
		2	.00000000000868588963789
		1	.00000000000434294481902
	12	9	.00000000000390865033713
		8	.00000000000347435585521
		7	.00000000000304006137332
		6	.00000000000260576689141
		5	.00000000000217147240950
		4	.00000000000173717792761
		3	.00000000000130288344573
		2	.00000000000086858896379
		1	.00000000000043429448190

* The ABACUS; an Instrument that works Natural Arithmetic, and makes Logarithms EXTERNALLY.

This RADIX, which enters the Twelfth Clafs at the Twenty-third Figure in Decimals, would have been perfect at the Twenty-fourth: but it will now conftruct Logarithms to Twenty-two Places. Out of this I take the following, which, according to my Plan, confifts of One Hundred Logarithms; and carries Logarithms to Twenty Figures true.

RADIX OF TEN CLASSES.

N. 10.		Log. 1.
0	9	.954242509439324874590
	8	.903089986991943585641
	7	.845098040014256830712
	6	.778151250383643632509
	5	.698970004336018804786
	4	.602059991327962390427
	3	.477121254719662437295
	2	.301029995663981195214
	1	.000000000000000000000
1	9	.278753600952828961536
	8	.255272505103306069804
	7	.230448921378273928540
	6	.204119982655924780855
	5	.176091259055681242081
	4	.146128035678238025926
	3	.113943352306836769206
	2	.079181246047624827723
	1	.041392685158225040750
2	9	37426497940623635200
	8	33423755486949702313
	7	29383777685209640835
	6	25305865264770240847
	5	21189299069938072793
	4	17033339298780354848
	3	12837224705172205171
	2	08600171761917561049
	1	04321373782642574275

	3	9	3891166236910521715
		8	3460532109506486157
		7	3029470553618007169
		6	2597980719908592312
		5	2166061756507676230
		4	1733712809000529768
		3	1300933020418118801
		2	0867721531226912493
		1	0434077479318640669
	4	9	390689249910131029
		8	3472966853635540688
		7	3038997848124918 10
		6	2604985473903468818
		5	2170929722302082822
		4	1736830584649188823
		3	1302688052270610004
		2	0868502116489572229
		1	0434272768626696637
	5	9	3908474458416739 2
		8	3474216888403320 1
		7	3039954976139869 4
		6	2605688721539547 9
		5	2171418124515513 7
1		4	1737143184980922 2
		3	1302863902848926 1
		2	0868580278032675 7
		1	0434292310445318 7
	6	9	3908632748308282
		8	3474341957876713
		7	3040050733157610
		6	2605759074150106
		5	2171466980853331
		4	1737174453266417
		3	1302881491388496
		2	0868588095218698
		1	0434294264756156

	7	9	39086485782376 7
		8	34743544654844 1
		7	30400603093017 8
		6	26057661096897 6
		5	21714718666483 4
		4	17371775801775 1
		3	13028832502772 8
		2	08685888769476 2
		1	04342944601885 3
	8	9	3908650161240 0
		8	3474355716251 8
		7	3040061266920 6
		6	2605766813246 5
		5	2171472355229 4
		4	1737177892869 4
		3	1302883426166 5
		2	0868588955120 6
		1	0434294479731 8
	9	9	390865031954 0
		8	347435584132 9
		7	304006136268 3
		6	260576688360 2
		5	217147240408 8
		4	173717792413 9
		3	130288344375 5
		2	086858896293 8
		1	043429448168 6
	10	9	39086503353 7
		8	34743558538 4
		7	30400613722 6
		6	26057668906 4
		5	21714724089 7
		4	17371779272 7
		3	13028834455 1
		2	08685889637 2
		1	04342944818 8

This

This RADIX of Ten Classes is extended to Twenty-one Places in the Logarithms, in order to secure twenty places both in Logarithms and Numbers.

The Approximation by Classical Numbers must be carried to Twenty-one places, consisting of Ten Nines and Eleven Defectives: Hence the *Line of Ratios* will also extend to Twenty-one places, in the Middle of which the CORRECTING *Ratio* stands: When this *Middle Ratio* is either 9, 8, 7, or 6, you must carry by way of CORRECTION, 16, 13, 10, or 7, respectively: But if the *Middle Ratio* be 5, 4, 3, 2, 1, or 0, then carry 5, 4, 3, 2, 1, or 0, respectively to the Twenty-first, or lowest place, when you begin to cast up the Logarithms. Thus every Logarithm constructed to Twenty-one Figures, will have Twenty Figures true.

Note. This easy Correction, which may be used either in making the Logarithm, or after it is made, is yet more valuable on another account; viz. It holds good every where, for a RADIX of Four, Five, Six, Seven, Eight, Nine, Ten, or any other Number of Classes, which may be made hereafter, ascertaining for every CLASS Two Figures in the Logarithm.

2. When the Logarithm required is within the limits of a RADIX, there needs no correction, because you can then go one place beyond your Design.

This RADIX of TEN CLASSES constructs the Logarithms to all Natural Numbers from Unity up to Twenty Figures in the Number; and also the Logarithms to the Natural Sines, Tangents, Secants, Versed Sines, &c. To Twenty Figures in the *Logarithm*; and *vice versa*.

RADIX

Radix of Six Classes.

	N. 10.		L. 1.
	0	9	.954242509439
		8	.903089986992
		7	.845098040014
		6	.778151250384
		5	.698970004336
		4	.602059991328
		3	.477121254720
		2	.301029995664
		1	.000000000000
	1	9	.278753600953
		8	.255272505103
		7	.230448921378
		6	.204119982656
		5	.176091259056
		4	.146128035678
		3	.113943352307
		2	.079181246048
		1	.041392685158
	2	9	37426497941
		8	33423755487
		7	29383777685
		6	25305865265
		5	21189299070
		4	17033339299
		3	12837224705
		2	08600171762
		1	04321373783
	3	9	3891166237
		8	3460532110
		7	3029470554
		6	2597980720
1		5	2166061757
		4	1733712809
		3	1300933020
		2	0867721531
		1	0434077479

4	9	390689250
	8	347296685
	7	303899785
	6	260498547
	5	217092972
	4	173683058
	3	130268805
	2	086850212
	1	043427277
5	9	39084745
	8	34742169
	7	30399550
	6	26056887
	5	21714181
	4	17371432
	3	13028639
	2	08685803
	1	04342923
6	9	3908633
	8	3474342
	7	3040051
	6	2605759
	5	2171467
	4	1737174
	3	1302881
	2	0868588
	1	0434294

This RADIX of SIX CLASSES (containing Sixty Four Logarithms) without Correction, constructs the TABLE of Lagarithms in *Sherwin* for Natural Numbers; as also his TABLES of Logarithmic Sines, Tangents, Secants and Versed Sines from the Natural. Also *Ulacq*'s CANON of *Ten Figure* Logarithms to Five Figures in the Number; with all the Intermediates to Ten Figures in the Number. Also *Dodson*'s Anti-logarithmic CANON of *Eleven Figure* Numbers to Five Figures in the Logarithm; with all the Intermediates to Eleven Figures in the Logarithm. And with the CORRECTION it extends to Twelve Figures in the Number and Twelve Figures in the Logarithm.

RADIX

Radix of 4 Classes.			
	N.	10.	L. 1.
	0	9	.95424251
		8	.90308999
		7	.84509804
		6	.77815125
		5	.69897000
		4	.60205999
		3	.47712125
		2	.30103000
		1	.00000000
	1	9	.27875360
		8	.25527251
		7	.23044892
		6	.20411998
		5	.17609126
		4	.14612804
		3	.11394335
		2	.07918125
		1	.04139269
	2	9	3742650
		8	3342376
		7	2938378
		6	2530587
		5	2118930
1		4	1703334
		3	1283722
		2	0860017
		1	0432137
	3	9	389117
		8	346053
		7	302947
		6	259798
		5	216606
		4	173371
		3	130093
		2	086772
		1	043408

	4	9	39069
		8	34730
		7	30390
		6	26050
		5	21709
		4	17368
		3	13027
		2	08685
		1	04343
			R. F. 1770.

This RADIX of Four CLASSES, containing but Forty Six Logarithms, forming a RULE of a portable Size, without the Correction, constructs all Dr. *Sherwin*'s Mathematical TABLES; viz. of Logarithms to Natural Numbers, Sines, Tangents, Secants, &c. as in the COROLLARY. And with the CORRECTION it extends to Eight Figures in the Number, and Eight Figures in the Logarithm.

FINIS.